(9) × $\cos\phi$ + (8) × $\sin\phi$ ⇒

$$\sin\phi\cos\phi\,\hat{i} + \sin^2\phi\,\hat{j} = \hat{s}\sin\phi$$

$$-\sin\phi\cos\phi\,\hat{i} + \cos^2\phi\,\hat{j} = \hat{\phi}\cos\phi$$

add

$$(\sin^2\phi + \cos^2\phi)\,\hat{j} = \hat{s}\sin\phi + \hat{\phi}\cos\phi$$

$$\boxed{\hat{j} = \hat{y} = \sin\phi\,\hat{s} + \cos\phi\,\hat{\phi}} \quad \text{——(11)}$$

eqⁿ (8) × $\cos\phi$ − (9) × $\sin\phi$

$$\cos^2\phi\,\hat{i} + \sin\phi\cos\phi\,\hat{j} = \cos\phi\,\hat{s}$$

$$-\sin^2\phi\,\hat{i} \pm \cos\phi\sin\phi\,\hat{j} = \sin\phi\,\hat{\phi}$$

sub.

$$\boxed{\hat{i} = \hat{x} = \cos\phi\,\hat{s} - \sin\phi\,\hat{\phi}} \quad \text{——(12)}$$

$$\boxed{\hat{z} = \hat{z}} \quad \text{——(13)}$$

Gradient, Divergance & Curl :-

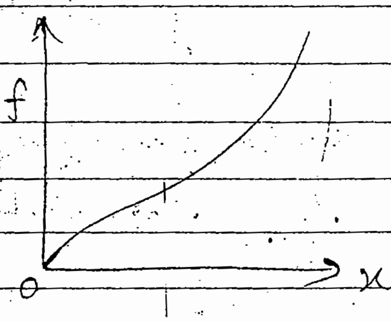
Gradient :- It is defined for a scalar. We can not take grad of any vector.

suppose a fucⁿ varry with x, $f(x)$

then $df = \underbrace{\left(\frac{\partial f}{\partial x}\right)}_{\text{slop}} dx$

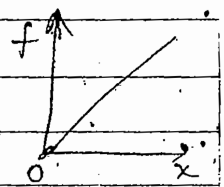
This derivative shows how f increases with x

This is the slop of the graph.



If graph linear then slop → const.

Non " → change



x → independent

f → dependent

classmate

Date

Page

indi: variable x shows → change in f by changing x.

Consider a funⁿ $T(x, y, z)$

Now T is changing with x, y, z. It can't describe by one variable.

Its differential

$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

We know that

$$\vec{\nabla} \equiv \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

So we can write

$$\boxed{dT = \underbrace{(\vec{\nabla} T)}_{\text{gradient of fun}^n\text{ (temp.)}} \cdot d\vec{r}}$$

→ gradient of a scalar is a vector quantity.

$$dT = \underbrace{|\vec{\nabla} T|}_{\text{magnitude of grad. of scalar}} dr \cos\theta$$

If $\theta = 0$,

then change is maximum

If $\theta = 90$, then change is minimum.

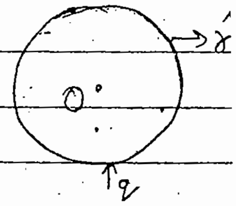
Dirⁿ of grad. of scalar shows dirⁿ of maximum increase of the funⁿ. ~~The dirⁿ~~

e.g. $\vec{E} = -\vec{\nabla} V$

dirⁿ of electric field is in the dirⁿ of maximum decrease of potential (bcz there is − sign)

Let us consider a conducting sphere on which electric field is normal to the surface.

It is equi potⁿ surface. So Elec. field is in the dirⁿ of max. dec. in potⁿ



$$\boxed{\bar{\nabla} T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}}$$ → In Cartesian

$$\boxed{\bar{\nabla} T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\phi}}$$ → In spherical polar

$$\boxed{\bar{\nabla} T = \left(\frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}\right)}$$ → In cylindrical

Now, <u>general formula for gradient</u> :-

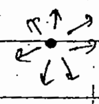
$$\boxed{\bar{\nabla}(r^n) = n r^{n-2}\vec{r} = n r^{n-1}\hat{r}}$$ $[\vec{r} = r\hat{r}]$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

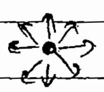
$\vec{r} = (x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}$

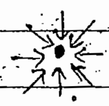
<u>Divergence</u> :- It is defined for a vector.
Divergence means spread (how much a vector can spread in space)
How much a vector diverges (spread) from a point in the space.

for source → +ve divergence
" sink → -ve "

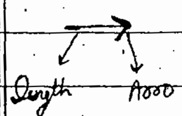


<u>Outward dir is +ve</u> <u>Inward is -ve</u>



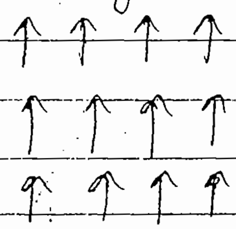


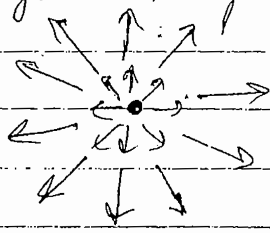
<u>Graphical repⁿ of Divergence</u>



, Arrow shows dirⁿ & length shows magnitude.

If vector length is same & no change in magnitude then zero divergence.

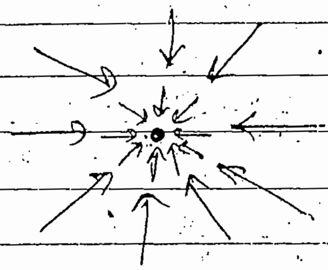




zero divergence

Non zero div. bcz length is changed.
↓
+ve div

No Variation in mag. → Zero div
Variation in mag → Non-zero div.



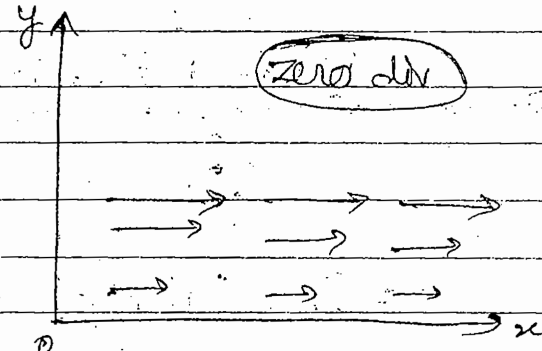
This is −ve divergence.
away from source → +ve
towards sink → −ve

If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

formula ⇒ $\nabla \cdot \vec{A} = \left(\frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z \right)$

Variation of x comp. of A with x.
" y " y.
" z " z.

This is zero divergence.
dir^n of vector is x
& change in mag. is in y-dir^n.
There is no chang in length along x-dir^n.



here $\nabla \cdot A = \frac{\partial}{\partial x} A_x$.

(Here Non-zero curl but zero div.)